Tabla de Identidades Trigonométricas

Funciones Trigonométricas

$$\tan x = \frac{\sin x}{\cos x}, \quad \sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}, \quad \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

Funciones Trigonométricas en función de las Otras Cinco

	$\sin x$	$\cos x$	$\tan x$
$\sin x =$	$\sin x$	$\pm\sqrt{1-\cos^2 x}$	$\pm\dfrac{\tan x}{\sqrt{1+\tan^2 x}}$
$\cos x =$	$\pm\sqrt{1-\sin^2 x}$	$\cos x$	$\pm\dfrac{1}{\sqrt{1+\tan^2 x}}$
$\tan x =$	$\pm\dfrac{\sin x}{\sqrt{1-\sin^2 x}}$	$\pm\dfrac{\sqrt{1-\cos^2 x}}{\cos x}$	$\tan x$
$\csc x =$	$\dfrac{1}{\sin x}$	$\pm\dfrac{1}{\sqrt{1-\cos^2 x}}$	$\pm\dfrac{\sqrt{1+\tan^2 x}}{\tan x}$
$\sec x =$	$\pm\dfrac{1}{\sqrt{1-\sin^2 x}}$	$\dfrac{1}{\cos x}$	$\pm\sqrt{1+\tan^2 x}$
$\cot x =$	$\pm\dfrac{\sqrt{1-\sin^2 x}}{\sin x}$	$\pm\dfrac{\cos x}{\sqrt{1-\cos^2 x}}$	$\dfrac{1}{\tan x}$
	$\csc x$	$\sec x$	$\cot x$
$\sin x =$	$\dfrac{1}{\csc x}$	$\pm\dfrac{\sqrt{\sec^2 x-1}}{\sec x}$	$\pm\dfrac{1}{\sqrt{1+\cot^2 x}}$
$\cos x =$	$\pm\dfrac{\sqrt{\csc^2 x-1}}{\csc x}$	$\dfrac{1}{\sec x}$	$\pm\dfrac{\cot x}{\sqrt{1+\cot^2 x}}$
$\tan x =$	$\pm\dfrac{1}{\sqrt{\csc^2 x-1}}$	$\pm\sqrt{\sec^2 x-1}$	$\dfrac{1}{\cot x}$
$\csc x =$	$\csc x$	$\pm\dfrac{\sec x}{\sqrt{\sec^2 x-1}}$	$\pm\sqrt{1+\cot^2 x}$
$\sec x =$	$\pm\dfrac{\csc x}{\sqrt{\csc^2 x-1}}$	$\sec x$	$\pm\dfrac{\sqrt{1+\cot^2 x}}{\cot x}$
$\cot x =$	$\pm\sqrt{\csc^2 x-1}$	$\pm\dfrac{1}{\sqrt{\sec^2 x-1}}$	$\cot x$

Algunos Valores Especiales

Función	$0(0°)$	$\frac{\pi}{12}(15°)$	$\frac{\pi}{6}(30°)$	$\frac{\pi}{4}(45°)$	$\frac{\pi}{3}(60°)$	$\frac{5\pi}{12}(75°)$	$\frac{\pi}{2}(90°)$
sin	0	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	1
cos	1	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	0
tan	0	$2-\sqrt{3}$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$2+\sqrt{3}$	$\nexists(\pm\infty)$
csc	$\nexists(\pm\infty)$	$\sqrt{6}+\sqrt{2}$	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	$\sqrt{6}-\sqrt{2}$	1
sec	1	$\sqrt{6}-\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	$\sqrt{6}+\sqrt{2}$	$\nexists(\pm\infty)$
cot	$\nexists(\pm\infty)$	$2+\sqrt{3}$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	$2-\sqrt{3}$	0

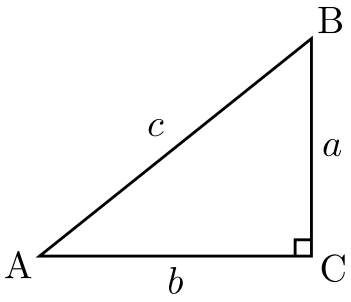
Identidades por Simetría, Periodicidad o Desplazamiento

$-x$ *o* $360° - x$	$90° - x$	$180° - x$
$\sin(-x) = -\sin x$	$\sin(\frac{\pi}{2} - x) = +\cos x$	$\sin(\pi - x) = +\sin x$
$\cos(-x) = +\cos x$	$\cos(\frac{\pi}{2} - x) = +\sin x$	$\cos(\pi - x) = -\cos x$
$\tan(-x) = -\tan x$	$\tan(\frac{\pi}{2} - x) = +\cot x$	$\tan(\pi - x) = -\tan x$
$\csc(-x) = -\csc x$	$\csc(\frac{\pi}{2} - x) = +\sec x$	$\csc(\pi - x) = +\csc x$
$\sec(-x) = +\sec x$	$\sec(\frac{\pi}{2} - x) = +\csc x$	$\sec(\pi - x) = -\sec x$
$\cot(-x) = -\cot x$	$\cot(\frac{\pi}{2} - x) = +\tan x$	$\cot(\pi - x) = -\cot x$
$x + 90°$	$x + 180°$	$x + 360°$
$\sin(x + \frac{\pi}{2}) = +\cos x$	$\sin(x + \pi) = -\sin x$	$\sin(x + 2\pi) = +\sin x$
$\cos(x + \frac{\pi}{2}) = -\sin x$	$\cos(x + \pi) = -\cos x$	$\cos(x + 2\pi) = +\cos x$
$\tan(x + \frac{\pi}{2}) = -\cot x$	$\tan(x + \pi) = +\tan x$	$\tan(x + 2\pi) = +\tan x$
$\csc(x + \frac{\pi}{2}) = +\sec x$	$\csc(x + \pi) = -\csc x$	$\csc(x + 2\pi) = +\csc x$
$\sec(x + \frac{\pi}{2}) = -\csc x$	$\sec(x + \pi) = -\sec x$	$\sec(x + 2\pi) = +\sec x$
$\cot(x + \frac{\pi}{2}) = -\tan x$	$\cot(x + \pi) = +\cot x$	$\cot(x + 2\pi) = +\cot x$

Cálculo de Funciones Trigonométricas

Función	Derivada	Integral
$\sin x$	$\cos x$	$-\cos x + C$
$\cos x$	$-\sin x$	$\sin x + C$
$\tan x$	$\sec^2 x = 1 + \tan^2 x$	$-\ln\lvert\cos x\rvert + C$
$\csc x$	$-\csc x \cot x$	$-\ln\lvert\csc x + \cot x\rvert + C$
$\sec x$	$\sec x \tan x$	$\ln\lvert\sec x + \tan x\rvert + C$
$\cot x$	$-\csc^2 x = -(1 + \cot^2 x)$	$\ln\lvert\sin x\rvert + C$

Ley de Senos



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Ley de Cosenos

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Ley de Tangentes

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$$
$$\frac{b-c}{b+c} = \frac{\tan\left(\frac{B-C}{2}\right)}{\tan\left(\frac{B+C}{2}\right)}$$
$$\frac{a-c}{a+c} = \frac{\tan\left(\frac{A-C}{2}\right)}{\tan\left(\frac{A+C}{2}\right)}$$

Identidades Pitagóricas

$$\cos^2 x + \sin^2 x = 1$$
$$\sec^2 x - \tan^2 x = 1$$
$$\csc^2 x - \cot^2 x = 1$$
$$\sin x = \pm\sqrt{1-\cos^2 x}$$
$$\cos x = \pm\sqrt{1-\sin^2 x}$$

Suma y Diferencia de Ángulos

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$
$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$
$$\csc(x \pm y) = \frac{1}{\sin(x \pm y)}$$
$$\sec(x \pm y) = \frac{1}{\cos(x \pm y)}$$
$$\cot(x \pm y) = \frac{\cot x\cot y \mp 1}{\cot y \pm \cot x}$$

Producto a Suma

$$\cos x\cos y = \cos(x-y) + \frac{\cos(x+y)}{2}$$
$$\sin x\sin y = \cos(x-y) - \frac{\cos(x+y)}{2}$$
$$\sin x\cos y = \sin(x+y) + \frac{\sin(x-y)}{2}$$
$$\cos x\sin y = \sin(x+y) - \frac{\sin(x-y)}{2}$$

Suma a Producto

$$\sin x \pm \sin y = 2\sin\left(\frac{x \pm y}{2}\right)\cos\left(\frac{x \mp y}{2}\right)$$
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$
$$\tan x \pm \tan y = \frac{\sin(x \pm y)}{\cos x\cos y}$$

Identidades de Ángulo Doble

$$\sin 2x = \frac{2\tan x}{1+\tan^2 x} = 2\sin x\cos x$$
$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x} = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$$

Identidades de Ángulo Triple

$$\begin{aligned}
\sin 3x &= 3\cos^2 x \sin x - \sin^3 x \\
&= 3\sin x - 4\sin^3 x \\
\cos 3x &= \cos^3 x - 3\sin^2 x \cos x \\
&= 4\cos^3 x - 3\cos x \\
\tan 3x &= \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} \\
\cot 3x &= \frac{3\cot x - \cot^3 x}{1 - 3\cot^2 x}
\end{aligned}$$

Identidades de Ángulo Medio

$$\begin{aligned}
\sin\tfrac{x}{2} &= \pm\sqrt{\frac{1-\cos x}{2}} \\
\cos\tfrac{x}{2} &= \pm\sqrt{\frac{1+\cos x}{2}} \\
\tan\tfrac{x}{2} &= \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \csc x - \cot x \\
&= \frac{\sin x}{1+\cos x} \\
\cot\tfrac{x}{2} &= \pm\sqrt{\frac{1+\cos x}{1-\cos x}} = \csc x + \cot x \\
&= \frac{\sin x}{1-\cos x} = \frac{1+\cos x}{\sin x}
\end{aligned}$$

$$\tan\left(\frac{x+y}{2}\right) = \frac{\sin x + \sin y}{\cos x + \cos y} = -\frac{\cos x - \cos y}{\sin x - \sin y}$$

Reducción de Exponentes

$$\begin{aligned}
\sin^2 x &= \frac{1-\cos 2x}{2} \\
\sin^3 x &= \frac{3\sin x - \sin 3x}{4} \\
\sin^4 x &= \frac{3 - 4\cos 2x + \cos 4x}{8} \\
\sin^5 x &= \frac{10\sin x - 5\sin 3x + \sin 5x}{16}
\end{aligned}$$

$$\begin{aligned}
\cos^2 x &= \frac{1+\cos 2x}{2} \\
\cos^3 x &= \frac{3\cos x + \cos 3x}{4} \\
\cos^4 x &= \frac{3 + 4\cos 2x + \cos 4x}{8} \\
\cos^5 x &= \frac{10\cos x + 5\cos 3x + \cos 5x}{16}
\end{aligned}$$

$$\begin{aligned}
\sin^2 x \cos^2 x &= \frac{1-\cos 4x}{8} \\
\sin^3 x \cos^3 x &= \frac{3\sin 2x - \sin 6x}{32} \\
\sin^4 x \cos^4 x &= \frac{3 - 4\cos 4x + \cos 8x}{128} \\
\sin^5 x \cos^5 x &= \frac{10\sin 2x - 5\sin 6x + \sin 10x}{512}
\end{aligned}$$

Diferencia de Cuadrados a Producto

$$\begin{aligned}
\sin^2(x) - \sin^2(y) &= \sin(x+y)\sin(x-y) \\
\cos^2(x) - \sin^2(y) &= \cos(x+y)\cos(x-y)
\end{aligned}$$

Composición de Funciones

$$\begin{aligned}
\sin(\arccos x) &= \sqrt{1-x^2} \\
\tan(\arcsin x) &= \frac{x}{\sqrt{1-x^2}} \\
\sin(\arctan x) &= \frac{x}{\sqrt{1+x^2}} \\
\tan(\arccos x) &= \frac{\sqrt{1-x^2}}{x} \\
\cos(\arctan x) &= \frac{1}{\sqrt{1+x^2}} \\
\cot(\arcsin x) &= \frac{\sqrt{1-x^2}}{x} \\
\cos(\arcsin x) &= \sqrt{1-x^2} \\
\cot(\arccos x) &= \frac{x}{\sqrt{1-x^2}}
\end{aligned}$$

Suma y Diferencia de Inversas

$$\begin{aligned}
\arcsin x + \arccos x &= \frac{\pi}{2} \\
\arctan x + \operatorname{arccot} x &= \frac{\pi}{2} \\
\arctan x + \arctan\frac{1}{x} &= \begin{cases} \frac{\pi}{2}, & \text{si } x > 0 \\ -\frac{\pi}{2}, & \text{si } x < 0 \end{cases}
\end{aligned}$$

$$\begin{aligned}
&\arcsin x \pm \arcsin y \\
&\qquad = \arcsin(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}) \\
&\arccos x \pm \arccos y \\
&\qquad = \arccos(xy \mp \sqrt{(1-x^2)(1-y^2)}) \\
&\arctan x \pm \arctan y \\
&\qquad = \arctan\left(\frac{x \pm y}{1 \mp xy}\right)
\end{aligned}$$